AN

INTRODUCTORY TEXT-BOOK

OF

INDUCTIVE LOGIC

AN

INTRODUCTORY TEXT-BOOK

OF

INDUCTIVE LOGIC

FOR THE USE OF JUNIOR STUDENTS

BY

P. K. RAY, D.Sc. (Lond. & Edin.

AUTHOR OF

"A TEXT-BOOK OF DEDUCTIVE LOGIC".

CALCUTTA:

S. K. LAHIRI & Co.,

56, COLLEGE STREET,

1916.

Printed at the Baptist Mission Press,
41, Lower Circular Road, Calcutta.

PREFACE

At the request of many Professors who teach the subject to the Intermediate classes in our Colleges, I have written this Introductory Text-book of Inductive Logic. I have tried to exclude from it all that would be unintelligible to those who begin to study the subject without any special scientific knowledge. The book presupposes only an elementary knowledge of Deductive Logic and an acquaintance with the phenomena of Nature, which a schoolboy acquires or may acquire in the ordinary course of his life. My object throughout has been to produce a book which would promote the study of a subject which has been rightly regarded as an Introduction alike to Science and Philosophy. I trust that the study of this book will be a good preparation for a student, for his subsequent course, whether it be Philosophy or Science.

The Introductory Text-book is completed in sixteen chapters specially designed for junior students. The Appendix contains additional matter for advanced students. In the Note (pp. 93–96) I have referred to certain points which should not, I thought, be entirely overlooked even in an Introductory Text-book.

My best thanks are due to Mr. K. C. Bhattacharjee, M.A., P.R.S., Professor of Logic and Philosophy in the Bethune College, Calcutta, who has made many valuable suggestions in the course of perusing the manuscript and correcting the proof-sheets. I should be obliged if the Professors who may use this book, would communicate to me their suggestions for its improvement.

P. K. RAY.

7, BALLYGUNGE CIRCULAR ROAD,
CALCUTTA,
April, 1916.

CONTENTS

CHAPTER I

CHAPTER II

PERCEPTION AND OBSERVATION

CHAPTER III

OBSERVATION AND EXPERIMENT

CHAPTER IV

CLASSIFICATION, DEFINITION, INDUCTION

CHAPTER V

DIFFERENT KINDS OF INDUCTION

CHAPTER VI

METHODS OF INDUCTION

CHAPTER VII

(2) METHODS OF ELIMINATION

CHAPTER VIII

METHOD OF AGREEMENT, AND JOINT METHOD OF AGREEMENT AND DIFFERENCE

I. Method of Agreement

II. The Joint Method of Agreement and Difference

CHAPTER IX

METHOD OF DIFFERENCE

CHAPTER X

METHOD OF CONCOMITANT VARIATIONS

CHAPTER XI

METHOD OF RESIDUES

CHAPTER XII

DEDUCTIVE METHOD

CHAPTER XIII

HYPOTHETICAL METHOD

CHAPTER XIV

EXPLANATION

CHAPTER XV

MILL'S DOCTRINE OF CAUSE

NOTE

CHAPTER XVI

FALLACIES

APPENDIX

DEFINITION AND PROVINCE OF LOGIC

INTRODUCTORY TEXT-BOOK OF INDUCTIVE LOGIC.

CHAPTER I.

Mill's View of Inductive Logic.

§ 1. Mill divides all truths, according to the way in which they are known, into (*i*) Intuitive and (*ii*) Inferential.

(*i*) Intuitive truths are known immediately without any reasoning. Mill regards the facts of consciousness—the mental feelings and bodily sensations—as known intuitively. "I am hungry now," "I was vexed yesterday," "I am touching this paper and seeing its whiteness," etc., are propositions which are known immediately without reasoning. They express certain facts of consciousness which are directly known and which are not inferred from other facts. According to Mill all such facts are particulars existing at a certain time.

(*ii*) Inferential truths are known mediately by reasoning. They are inferred from intuitive truths by the processes of Naming, Definition, Classification, Generalisation, etc., which form the subject-matter of Logic From the particular facts known by Intuition, a general proposition may be inferred according to the Canons laid down by the Logic of Induction. According to Mill we usually reason from some particular facts to some others with which they are known by experience to be

connected The universal type of reasoning, according to him, is that if a particular or individual thing, A, resembles another particular or individual thing, B, then what is true of A will be true of B, the certainty of the inference depending on the degree of resemblance between A and B. What is true of A and B will be true of any other particular or individual thing which resembles A and B in the same attribute or attributes in which they resemble each other. A, B, C, etc. are particulars resembling one another, say in the attribute m; then if the attribute n is known by experience to be connected with m and true of the particular A, n will be true of the particular B or C, or any other possessing the attribute m. A, B, C, etc. will thus form a class and give rise to a general proposition affirming the connection of the attribute m with n, that is, any particular or individual thing possessing the attribute m will also have the additional attribute n. If we call this class S with m as its common attribute or connotation, we may state the general proposition in the usual form, "All S is P" where 'P' is an adjective signifying the attribute n. For example, in the general proposition "All men are mortal," 'humanity' and 'mortality' correspond respectively to m and n, the particular things are individual men having the attribute, humanity, in common, with which the attribute, mortality, is known by experience to be connected. The process by which the universal proposition, "All men are mortal," is established from the particular cases of the death of individual men is called Induction and the operation by which it is applied to any living man or men is called Deduction by Mill.

§ 2. According to Mill, all universal propositions which are not merely verbal (analytical), or deduced from verbal propositions, are generalisations from particulars known by experience. It is the business of Inductive Logic to lay down the conditions to which we must

conform in order that a general proposition inferred from one or more particulars, may be true.

Mill holds that the Laws of Thought, the Axioms of Geometry etc., which are usually regarded as self-evident and intuitive, are also generalisations from experience. According to him, intuition gives us no knowledge of any universal principle. It only reveals to us the particular facts of consciousness. In our ordinary perceptions and observations there are two elements, (1) Intuitive, or the elements of original sensations or pure perceptions of the several senses, and (2) Inferential, or the elements added to them by reasoning. In order that our perceptions and observations may be true, the reasoning involved in them must conform to the conditions of correct reasoning. It is evident, therefore, that Mill is bound to analyse our ordinary perceptions and observations and distinguish the inferential elements in them from those that are primary and intuitive. Our ordinary perceptions and observations are, he points out, the results of reasoning and should be distinguished from the pure sense-perceptions which are intuitive. As the end of Logic, according to Mill, is the attainment of real or scientific truth, it is necessary that the data supplied by observations should be true and that the inferential elements contained in the observations should conform to the laws of Logic. He holds that there are no rules of Logic regulating pure perceptions or pure observations, that is, the intuitive elements involved in our ordinary perceptions and observations. They form the ultimate data of all reasoning. They depend on our physical and mental constitution and on the environment. Logic assumes them as they are in normal human beings.

No syllogistic reasoning is possible unless there are general or universal propositions. In the first figure to which all syllogisms may be reduced, the major premiss must be universal. Mill holds that as all universal propo-

sitions are generalisations from experience, no valid (true) syllogistic reasoning is possible unless we have valid generalisations. We may assume them or they may be analytical, but the conclusions will then be hypothetical and not true. In order that the conclusion of a syllogism may be true, the premisses must be true; and of these one, the major, being universal must be a generalisation from particulars known by experience. The problem for Mill's Logic is, therefore, to lay down the conditions to which a generalisation must conform in order that it may be true.

§ 3. Mill finds a solution of this problem in what he calls the Principle of the Uniformity of Nature. This principle is itself a generalisation from experience. By "Nature" he means the whole of phenomena, physical or mental, which are known or capable of being known by us. By Uniformity he means that there are constant relations or connections among these phenomena. He points out that the uniformity of nature consists of many uniformities in different departments of Nature. He classifies them under the three heads of (1) Coexistence, (2) Succession, and (3) Likeness. The phenomena of Nature coexist in space, succeed each other in time, and resemble one another in certain qualities, according to laws which are investigated in different sciences. Under Succession he recognises a special kind of uniformity, which he distinguishes from others and generalises under the Law of Causation. This Law means, according to Mill, (1) that every phenomenon has a cause, and (2) that the cause of a phenomenon is a phenomenon or an aggregate of phenomena, which invariably and unconditionally produces the phenomenon. Mill shows that the Law of Causation and the Principle of Uniformity of Nature form the basis of all scientific investigations. The observations and experiments in science are conducted on the assumption of their truth. The generalisations in Physics and Chemistry and other

Sciences are so many different instances of the two fundamental principles of Causation and Uniformity of Nature.

§ 4. According to Mill, an Induction is a valid generalisation from some to all particulars of a class. By a "particular" he means an individual belonging to the class. "Socrates, Plato, Aristotle and others have died; therefore all men will die" is an Induction Is it valid? It is valid if it conforms to the principle of the Uniformity of Nature—if a constant connection can be shown to exist between the attribute or attributes common to Socrates, Plato, Aristotle and others, i e. to men who have died, and the attribute of dying or death. What is the attribute in them which is so connected with death? They all have an organised body; and it must be shown by Inductive Methods that there is a constant connection between organisation and death—that wherever an individual has an organised body, it has in it also the elements leading to the dissolution of the organisation (connection of Coexistence) or that the phenomena constituting an organised body will be invariably followed by the phenomena of its dissolution (connection of Succession or Causation). The induction "All men will die" or "All men are mortal" may be established by the Method of Simple Enumeration. It may be shown that all individual men in past ages died, that many men in the present age have died, that there are no cases of men living beyond a certain age, that, so far as our experience goes, all men die after a certain age and that therefore all men who are now living and who may be hereafter born will die, i e., all men are mortal. The Method of Simple Enumeration is the most popular method of induction as it is also the most ancient. "Food nourishes," Water quenches thirst," "Fire burns," "Liquids expand by heat," "Bodies fall to the earth," "Crows are black," "Swans are white," "Plants have flowers," "Animals have power of locomo-

tion," "Metals conduct heat," etc., are inductions established by this method. Mill recognises it but points out that as it cannot establish the relation of cause and effect, the method is not of much use for scientific purposes For a valid scientific induction it is necessary to show that the subject and the predicate are causally connected—that the attribute connoted by the subject, or the phenomenon signified by it, is invariably and unconditionally followed by the attribute connoted by the predicate, or the phenomenon signified by it. In the inductions given above such a connection has not been shown to exist. In fact we are not sure that some of them are valid inductions. They are no doubt generalisations from our experience. But a larger experience may or may not confirm them. As a matter of fact some of them are not universally true. There are black swans; there are plants that have no flowers; there are animals without the power of locomotion In the case of the others it is necessary to state definitely the conditions under which they are true Mill, therefore, distrusts the Method of Simple Enumeration for scientific purposes and formulates certain methods which he calls the "Methods of Experimental Inquiry" and which are known as Mill's "Methods of Induction."

The Induction "All men are mortal" may be established by showing that there is a causal relation between the phenomena of human life and those of death For this purpose it would be necessary to analyse both the groups of phenomena into their constituent elements. The phenomena of human life include those of animal life; and the latter those of an organised living body; and it may be shown that the phenomena of organisation are invariably followed by those of the dissolution of the organism. An organism consists of certain organs performing the function of life It may be shown that the organs are so constituted and the natural environment is such that after a certain period they invariably cease

to work, and thus bring about the death of the organism; that is, there is a causal connection between the phenomena constituting the life of an organism and those constituting its death If such a connection can be established, the induction will be a valid one, according to Mill. The other popular inductions given above will be valid if a causal connection can be established between the subject and the predicate of the propositions, that is, between the phenomena signified by them. Mill's Experimental Methods of Inquiry are methods for discovering and proving such connections among the phenomena of Nature.

CHAPTER II.

Perception and Observation.

§ 1. Ordinary observations are inferences from direct perceptions

Inductive Logic is concerned with the truth of phenomena known by inference, as distinguished from what is known by intuition. The ultimate data of all inference are facts known by intuition. There is great difference among philosophers as to what is exactly known by intuition. We have seen that, according to Mill, intuition gives us a knowledge only of our mental feelings and bodily sensations. A sensation is a mental state produced by an external object. A perception is the reference of the sensation to the external object and implies, according to him, an act of inference According to others, a perception gives us a direct and immediate knowledge of an external object. The word perception is sometimes used in a wider sense, including our immediate knowledge of the internal subject. The latter is called Internal Perception as distinguished from the former which is called External Perception. Perceptions are also distinguished as original and immediate from those which are acquired and mediate. The former are intuitive while the latter are inferential. In our ordinary perceptions and observations, there is a combination of both the elements.

" What we are said to observe," says Mill, " is usually a compound result of which one-tenth may be observation, and the remaining nine-tenths inference "—Bk. IV, Chap. I, section 2. By 'observation' he means here an immediate perception as distinguished from an acquired

or mediate perception in which there are elements of inference. "In almost every act of our perceiving faculties, observation and inference are intimately blended" "I affirm, for example," says Mill, "that I hear a man's voice. This would pass, in common language, for a direct perception. All, however, which is really perception, is that I hear a sound. That the sound is a voice, and that voice the voice of a man, are not perceptions but inferences I affirm, again, that I saw my brother at a certain hour this morning. If any proposition concerning a matter of fact would commonly be said to be known by the direct testimony of the senses, this surely would be so. The truth, however, is far otherwise I only saw a certain coloured surface; or rather I had the kind of visual sensations which are usually produced by a coloured surface; and from these as marks, known to be such by previous experience, I concluded that I saw my brother. I might have had sensations precisely similar, when my brother was not there. I might have seen some other person so nearly resembling him in appearance, as, at the distance, and with the degree of attention which I bestowed, to be mistaken for him. I might have been asleep, and have dreamed that I saw him; or in a state of nervous disorder, which brought his image before me in a waking hallucination. In all these modes, many have been led to believe that they saw persons well known to them, who were dead or far distant. If any of these suppositions had been true, the affirmation that I saw my brother would have been erroneous; but whatever was matter of direct perception, namely, the visual sensations, would have been real. The inference only would have been ill-grounded. I should have ascribed those sensations to a wrong cause."

§ 2. Immediate Perceptions.

Each sense has its immediate or direct perceptions. Sight, for instance, gives us a knowledge of the colours

of objects; Hearing of the sounds produced by them: Active Touch, of the resistances offered by them and of their forms and sizes; Smelling, of their odours; Tasting, of their flavours. The colour, for example, of an orange is known by Sight, its form, size and solidity by Active Touch, its scent by Smelling, its flavour by Tasting. These are direct or immediate perceptions of an orange by the several senses. There is much difference of opinion among psychologists as to the exact knowledge supplied by each sense. All would, I think, agree that the original sensations of each sense constitute its direct perceptions of an object and that the qualities of the object are founded on these sense-perceptions. They would differ in their account of what is implied by each sense-perception. Is an object implied by every sense-perception? If so, what is its nature? Is it implied by some sense-perceptions only? If so, what are they? Does the perception of colour and sound imply any perception of extension? How is an object as a source of resistance and as occupying space known? Is it an inference or an intuition? A thorough discussion of these questions belongs to Psychology and Metaphysics.

§ 3. Mediate Perceptions.

A mediate perception of an object is an inference from an immediate perception of it. The inference of the taste of an orange from its appearance is a mediate perception. The inference of the distance of an object from its direct visual perceptions is a mediate perception acquired by experience So are also the perceptions of the form and size of an object at a distance from the sensations of sight or rather from the perceptions of its visual figure and size. The immediate perceptions of one sense become associated by experience with the immediate perceptions of another sense; and the one set may be inferred from the other. The inferences will be true or false according as they do or do not conform to the rules of Logic.

§ 4. Internal Perception.

In Internal Perception there is no special organ of sense; but the mind directly perceives its feelings, its desires, its ideas, its activities, etc. It is sometimes called Introspection, i.e., looking within. It is also called Self-consciousness, i.e., the consciousness of the self as distinguished from the not-self. Self-consciousness is direct and immediate knowledge of the phenomena of one's own mind. The mental phenomena are so connected with one another that it is possible to draw inferences as in the case of the phenomena of the external world. From the immediate consciousness of a particular feeling may be inferred the desire that is likely to follow. From the immediate consciousness of an idea may be inferred the feeling that accompanies or is asso ciated with it and the volition that is likely to follow. From a knowledge of the connections of the phenomena in one's own mind, inferences may be drawn in regard to the phenomena of another person's mind; and these may be verified by his gestures and actions or by direct reference to his consciousness.

As in the case of External Perception, Philosophers differ in their account of Internal Perception. The chief point of difference in the latter case is as regards the nature of the Self or the Mind. Does it merely consist of the phenomena or is there a unity or a permanent reality which is the ground of the phenomena and which is directly known along with them?

§ 5. The word perception has a still wider import than has been noticed above. It means also a direct knowledge of the relation between two given things. For example, the consciousness of the likeness or unlikeness of two things, say, in regard to their colour, is also an immediate perception. Likewise, the equality or inequality of two lines placed side by side is known by direct perception, i.e., intuitively. The identity or difference of two things in respect to any particular attribute is

known by immediate perception. The relation of co-existence or of succession between two given phenomena is perceived directly without any reasoning. It is held by some philosophers that the knowledge of the relation of A to C from the knowledge of the relation of each to B is also direct and immediate knowledge, i.e. an intuition, as in the case of the first axiom: $A=B$; $C=B$ $\therefore$ $A=C$. This is true if the three things remain unchanged, but will not be true in a world of flux. It is true of our ideas and concepts which are assumed to remain unaltered. In the case of phenomena, $A=C$ is an inference which must be verified by experience.

§ 6. The word perception as used in the phrases, Aesthetic Perception, Moral Perception, Spiritual Perception, etc., implies also direct and immediate knowledge of special kinds of facts and phenomena. The phenomena of beauty in the external and in the internal world are directly perceived by the mind. So are also the facts and phenomena of our moral and spiritual nature. Inductive Logic has to recognize the truth of all immediate and direct perceptions and lays down rules according to which true inferences may be drawn from them.

§ 7. The faculty of immediate perception, like all other faculties, grows by exercise. The child at first perceives all colours as more or less alike. But gradually he distinguishes them and shows his fondness for the bright colours. The colour-sense in an adult is more developed than in a child. Persons who are dealing in colours can easily distinguish the different shades of a colour, which, to an ordinary person, would appear to be the same. All this knowledge of the different colours and of their various shades is due to the growth of the faculty of visual perception. It is immediate and direct to the person in whom the faculty is developed. It is not the result of inference but of intuition. No amount of reasoning could produce in a person the knowledge of the difference between red and blue. This know-

ledge is intuitive and is only attained by a direct and immediate perception. The progress in intuitive knowledge caused by the growth of the faculty is true, not only of external perception, but also of the other kinds of perception noticed above. The facts directly known by the fully developed faculty of perception of every kind have to be recognised in Inductive Logic.

§ 8. Perception gives us immediate knowledge of what is present. Memory gives us immediate knowledge of what is past but has been perceived by us. The knowledge supplied by memory of past events and occurrences is as direct as the knowledge given by perception of what is happening now. It is true that memory sometimes becomes vague and faint and is not always reliable. It may require to be revived by circumstances; but, when thus revived, it recalls a past event with wonderful faithfulness. The facts recalled by memory must also be recognised by Inductive Logic.

CHAPTER III.

Observation and Experiment.

§ 1. Observation is the extension of immediate perception by inference. Experiment is the extension of observation by special arrangements of natural objects. By 'observation' we investigate the properties and laws of objects and phenomena as they occur in nature. By 'experiment,' objects are isolated from their natural combinations in nature, and phenomena are artificially produced for observation. Observation and experiment are necessary for obtaining the facts and phenomena of a science. The laws of the science are based upon these facts and phenomena. In Physics and Chemistry, for example, the general properties of material bodies, the special properties of liquids, solids and gases, the laws of expansion of these bodies by heat, the laws of chemical combination, the properties of the chemical elements and their various compounds, etc., have been determined by observation and experiment. In some sciences as Botany, Zoology, and Mineralogy, observation is the chief method of obtaining the facts, while in Chemistry and Physics, experiment is absolutely necessary for determining even the most elementary laws of these two sciences. It is by experiment that oxygen is prepared from its compounds in nature and its properties are determined. It is by experiment that the properties of the compounds which oxygen forms with the other elements such as carbon, nitrogen, etc., have been ascertained. The law of falling bodies has been determined by an experiment in which two bodies such as a piece of gold and a feather are allowed to fall through a glass cylinder which

has been emptied of its air. In such an experiment it is found that the two bodies fall to the bottom of the cylinder at the same time. The laws of reflection and refraction of light have been determined by experiments.

The truth of the laws of a science depends upon the truth of the facts and phenomena determined by observation and experiment. Observation and experiment must therefore be carefully conducted and the results accurately recorded for confirmation and future reference. For this purpose scientific apparatus as well as a complete system of terms are necessary—the former for producing and observing the phenomena and the latter for accurately recording them in language.

§ 2. Experiment has certain advantages over observation. When a phenomenon is produced by an experiment, we know all the circumstances under which it is produced. We can vary them as we like and notice the change in the phenomenon. We may thus determine a quantitative relation between the cause and the effect. But we can not produce a phenomenon by an experiment unless we know its cause. It is by observation that we can find out the probable cause of a given phenomenon. We observe many instances of it as it occurs in Nature and draw from them some inference as regards its probable cause. We may then experiment upon this cause and try to produce the phenomenon. The inference will be correct if the phenomenon is produced by it. But as it often happens the cause inferred may not produce the phenomenon. It may be only a part of the true cause and will not produce the phenomenon by itself. Or it may consist of the cause and other circumstances which are irrelevant to the production of the phenomenon. For the elimination of these irrelevant circumstances experiment is necessary. Take the phenomenon of vapour in the atmospheric air. What is its cause? We may observe many cases of the presence of vapour in the air on different days and in

different seasons. Its quantity increases in the rains, when the air becomes very humid. It is much less in the winter. It is more present in the air of Lower Bengal than in that of the Up-country. From these instances it is difficult to determine the conditions under which it is produced and the conditions under which its quantity in the air varies in different seasons and in different countries. It may be inferred that it is produced somehow from the water-sources of the country and that its quantity depends on the proximity to rivers and seas and on the prevalent winds, being greater, for example, during the monsoons. Its quantitative connection with the temperature of the air has been determined by experiment. It has also been determined by experiment that not only does water rise as vapour through heat as in the summer, but that all water-sources are giving off vapour throughout the year.

§ 3. In Inductive Logic it is usual to represent objects and their phenomena by symbols such as the letters of the alphabet. An object as consisting of a number of attributes, is represented by a combination of letters, each standing for an attribute. The attributes of an object are founded upon the phenomena of sense, which it produces in us. The attributes of a piece of chalk, for example, such as resistance (or solidity), figure, size, colour, etc. are grounded on the corresponding sense-perceptions. All the phenomena of objects are thus, on the one side, related to the mind and, on the other, to the object. They imply a sensibility of the former and a capacity of the latter to produce a sensation which is the content of the sense-perception. A piece of chalk is thus an aggregate of the various phenomena on which its attributes are grounded. The attributes are the capacities or potentialities of the chalk to produce certain sense-phenomena in us. It is these phenomena which form the subject-matter of science. The phenomena are represented by the same letters that stand for

the corresponding attributes. All phenomena may be traced to the attributes of an object or of a subject or mind The actions of one object upon another, or of one mind upon another, or of an object and a mind upon each other are phenomena implying attributes and may be represented by letters A simple phenomenon is represented by a single letter and a complex or compound phenomenon by two or more letters. The terms "simple" and "compound" as applied to phenomena are relative. A piece of chalk is sometimes taken as one phenomenon and represented by a single letter and sometimes as a compound one represented by several letters, standing for the several sense-phenomena of resistance, figure, size, colour, etc. it produces in us.

In Inductive Logic the antecedents of a phenomenon or both the antecedents and the consequents in a phenomenon are represented by letters. In the latter case it is evident that the phenomenon is a very complex one, consisting of certain antecedent and certain consequent phenomena, each of which may again be complex. In the former case the phenomenon may be simple or complex. Even when we speak of the antecedent and the consequent in the singular, the phenomenon referred to may be complex.

CHAPTER IV.

Classification and Definition.

§ 1. By immediate perception, observation and experiment, we know individual things and phenomena; and the problem of classification is their grouping according to their resemblances and differences. They may be divided into two broad classes according as they are known by external or by internal perception. Those known by external perception occupy space and those known by internal perception occur in time and do not occupy space. All external things and phenomena thus form a very large class, having the attribute of being in space as their common quality, while all internal or mental things and phenomena form another very large class distinguished from the former by their not being in space. Both agree in being in time—things existing in time, and phenomena being changes in time.

All external things and phenomena may next be divided into classes according to the sense by which they are known. Thus there would be as many classes as there are senses giving us knowledge of external objects. Touch, sight and hearing are the most important knowledge-giving senses and there would thus be three classes, namely (1) Tangible, (2) Visible and (3) Audible. But these classes would not be exclusive of one another as one and the same individual thing may be an object of all the three senses. The classification would be one of attributes founded on conscious phenomena rather than of individual things. The latter may be classified according to the degree of resistance offered by them into (1) Solid, (2) Liquid, (3) Gaseous, (5) Ethereal. The first

three are known to have the element of resistance in different degrees, but the last—the medium of light, heat and electricity—appears to be without it or to have so little of it that the present scientific instruments have not been able to detect it.

All things offering resistance and occupying space may be next classified into those that have life and those that are without it, namely, (1) living (plants and animals) and (2) non-living (minerals, rocks, stones, chemical elements and compounds, etc.) Living things may be classified according as they have sensibility and power of locomotion or not, into animals and plants. Animals may be classified according to their organisation, i.e. the degree of life developed in them. So may also be plants. The classification of plants is the subject of Systematic Botany and that of animals of Zoology.

The first broad classification of plants is into (1) flowering, i.e. those that have flowers, and (2) non-flowering, i.e. those that are without any flowers, e.g. ferns, mosses, etc. The first broad classification of animals is into (1) Vertebrata, i.e. those that have the backbone, and (2) Invertebrata, i.e. those without it, e.g. insects, molluscs, etc. One of the earliest classification of animals is according to their place of living—(1) land, (2) water, or (3) both (amphibious).

§ 2. The object of all these classifications is to bring together those things that resemble one another and to separate them from those from which they differ. The greater the number of qualities in which the individuals of a class resemble, and the more important these qualities, the more scientific is a classification. The importance of a quality depends on its persistency and its connection with other qualities in a group of individuals. The vertebral column is an important attribute for the classification of animals because it is present in all higher animals including fishes; and it is further found that it is correlated with a number of organs such as the skull,

the spinal cord, the heart, the brain, the organs of sense, etc. in an animal. Likewise the presence or absence of flowers is a very important attribute of plants. All flowering plants have many other common characters, while all flowerless plants are devoid of them, but have other common characters. A scientific classification takes into consideration all the attributes of the things to be classified and groups them according to their most important characters. A classification is said to be artificial if it is founded on one character for a particular purpose. If, however, the character is an important and persistent one, an artificial classification may pass into a scientific one. The classification of objects founded on the single quality of resistance and its varying degrees is a scientific one.

§ 3. Classification is a very important process in science. It gives rise to classes, each possessing certain common attributes. The common attributes constitute the connotation of the class-name. New individuals may be included in the class if they are found to possess the common attributes. The class metal arises from a classification of elements into (1) metals and (2) non-metals and is characterised by certain qualities. If a new element is discovered and is found to possess some of those qualities, it is inferred that it possesses also the others, and the element is referred to the class metal. The inference may be verified afterwards by experiment. Likewise if a new plant is discovered and found to possess some of the characters of a particular class, it is referred to the class under the belief founded on the Uniformity of Nature, that the other characters, for example, the fruit, will also be found in due course. The new plant may be identified by its flower; and the fruit characteristic of the class may be produced in proper season.

New attributes may be added to the common attribute or attributes of a class with the increased knowledge of

the things belonging to the class. With the progress of Physics and Chemistry there has been an addition to our knowledge of the common attributes of the class metal. Similar change has taken place in the connotation of the class-names, plant, animal, living body, etc. Our knowledge of the things denoted by them has increased with the progress of Biology, Botany and Zoology, and it has become necessary to re-model their classifications on new principles. Of these the most important principle is that of Evolution of living things from a few original forms. The old classifications of plants and animals are being transformed into genealogical trees.

§ 4. The preceding section shows the close connection between Classification and Definition. The definition of a class-name consists of the attributes common to the individual things belonging to the class. The definition will increase in content with the advance in our knowledge of the common attributes. Definitions of the names will change as the classification is adapted to our increasing knowledge of the things classified. The definitions of metals, plants, animals, living bodies, etc., have changed with our increased knowledge of the things. The more accurate and advanced is our knowledge of the things defined, the truer is the definition. In defining the term metals, the object at first is to find out its correct meaning in popular usage, then its meaning in scientific usage. We shall thus have a popular and a scientific definition of the term. But the philosophic problem of definition of metals goes deeper. It attempts to find out the most fundamental qualities which are inherent in the known metals, which distinguish them from other substances, and from which their other qualities are derived. This may not be the case with the present scientific definition of the term. A final and complete definition implies a final and complete knowledge of the individual substances denoted by the term. It is therefore an ideal at which Science aims in its investigation

of truth rather than an accomplished fact in regard to any class of natural objects.

§ 5. It is evident that classification presupposes (1) a system of names for describing the parts and actions of individual things; (2) a system of names signifying the qualities in which the individual things resemble one another; (3) a system of names for the classes which arise from the grouping of the individual things according to their resemblances and differences.

The first and the second have been called Terminology and the third Nomenclature.

In describing a plant, for example, names or words are required for its various parts and organs such as the roots, the trunk, the branches, the leaves, the flowers, the fruits, etc., and for noting the various modifications of these parts and organs. These names must have definite meanings and be sufficient in number to describe accurately and fully the various parts of an individual plant so as to be able to distinguish it from another. The parts and organs of one plant resemble those of another; and names are required to note these common qualities. These two systems of names constitute what is called Terminology. When the plants observed and described are grouped into classes according to their resemblances and differences, names are selected for these classes according to their position in a scheme of classification. These names, especially those standing for the lowest classes called infimæ species, constitute what is called Nomenclature. Botany, Zoology, Mineralogy, Chemistry have each its systems of Terminology and Nomenclature.

§ 6. Generalisation and Induction.

Generalisation is the process by which what is known to be true of one or more individuals of a class is inferred to be true of the whole class. For instance, it is observed that water and oil are solidified by cold and it is inferred that all liquids are solidified by cold. The general proposition which is the result of a generalisa-

tion is called Induction. Whether the induction "all liquids are solidified by cold" is true or not will depend on further observation. If it cannot be verified by observation, recourse may be had to experiment. The liquid bodies that are not seen to solidify under the natural conditions of the cold weather may be solidified by cold produced artificially by properly devised experiments. The general proposition may be thus established by observation and experiment. It may be true of all the known liquids. But it may still be contradicted by some newly discovered liquid. In order that it may be established as a universal proposition, true of all liquids whatever, known or unknown, it is necessary to establish some connection between liquidity and cold on the one hand and solidity on the other, which will hold good under all conditions. Until such a connection is established, the generalisation must be regarded as empirical and may be falsified by a single instance to the contrary.

CHAPTER V.

The Different Kinds of Induction.

§ 1. Every valid induction is founded, according to Mill, on a constant relation or connection among the phenomena of Nature. He distinguishes three kinds of Induction corresponding to the relations of (1) Coexistence, (2) Likeness or Resemblance, (3) Succession.

(1) Inductions of Coexistence.

Some samples of gold have certain qualities; therefore, gold, wherever it may be found, will have the same qualities. This is an induction of coexistence. The qualities are connected with one another and coexist in every piece of gold. The connection among them is assumed to be a constant one on the ground of the Uniformity of Nature and is verified by experience. There are similar inductions founded on the coexistence of certain qualities in some samples of other chemical elements as also of chemical compounds and in some individuals of a class or species of plants or animals. To the class of Inductions of Coexistence belong all Laws of Nature which cannot be brought under the Law of Causation. The law that whatever has inertia gravitates is an Induction of Coexistence. No causal connection has been shown to exist between inertia and gravitating.

§ 2. (ii) Inductions of Likeness or Resemblance.

Every Induction of Coexistence is also an Induction of Likeness or Resemblance. The induction "all gold has certain qualities" may be stated in the form "all samples of gold resemble one another in those qualities." The induction "all material bodies gravitate" is the same as "all material bodies resemble one

another in the attributes of inertia and gravitating." The induction "all men are mortal" expresses the fact that all men resemble one another in the attributes, animality and mortality. Resemblance may vary in degree. In some cases it amounts to identity as in the samples of chemical elements and compounds. The different samples of oxygen, hydrogen, water, carbonic acid, silver, copper, all in their chemically pure state, have identical properties. What is true of one pure sample of a chemical element or compound is true of all pure samples of the same element or compound, because they resemble one another to such an extent that one sample cannot be distinguished from another. These inductions are founded on the uniformity of resemblance or identity between the different samples of the same substance. In other cases the resemblance is partial. It exists in the midst of difference. One man resembles another in certain qualities and differs from him in certain other qualities. What is therefore true of some men cannot be inferred of all men, unless what is inferred is causally connected with, or follows from, the qualities in which they resemble. For the quality inferred may be one which is connected with those qualities in which they differ from one another. It may be a special quality of a particular individual or of a particular race and may not be possessed by another individual or another race. Inductions founded on resemblance must therefore be carefully tested and brought under the Law of Causation.

§ 3. Inference from Analogy.

Inferences founded on the resemblance of two individuals to each other are distinguished as analogical. The earth resembles the planet Mars in certain qualities : it is therefore inferred by analogy that as the earth is inhabited by plants and animals, Mars is likewise inhabited by them. Such an inference will be only probable unless a causal connection can be established between the com-

mon qualities of the earth and Mars and the phenomena of plants and animals. Two brothers resemble each other in a number of qualities. What is true of one may be inferred of the other, if the quality inferred is connected with the qualities in which they resemble. The connection may be one of causation between the common qualities and the quality inferred; or the latter may follow from the former by way of deduction. Otherwise the inference will be only probable.

Analogical inferences which cannot be brought under the Law of Causation depend on the Laws of Coexistence and Resemblance. Two individuals, A and B, resemble each other in a number of attributes, p, q, r, s. A has an additional attribute t. That is, the attributes p, q, r, s, t are found coexisting in A. If t is causally connected with the other attributes, t will be present also in B. The inference will then be certain and amount to an induction, being true not only of B but also of any other individual belonging to the class of A and possessing the attributes p, q, r, s, t. If there is no causal connection between t and p, q, r, s, the inference will be probable, the degree of probability depending on the nature and number of the qualities in which B resembles A. If p, q, r, s are fundamental qualities of A and B, then t, which is found coexisting with them in A, will most probably be found coexisting with them in B. If p, q, r, s, are only superficial qualities, then the probability of t being found coexisting with p, q, r, s in B because it occurs with them in A will be much less. We have examples of such inferences in Zoology and Botany. A fish resembles an amphibian (say a frog) in the possession of a backbone, a spinal cord and brain, and a heart; the amphibian has organs of sense: the fish has therefore the organs of sense. The former has limbs; the latter has therefore limbs. The inferences here are analogical and should be verified by observation. The inference "the fish has organs of sense" similar to those of the

amphibian is found to be true, while the inference it has limbs is not quite true, because instead of limbs like those of the frog it has fins which correspond to them and perform the function of locomotion, but are not like them in form and structure. The fossilised remains of an extinct animal resemble a living animal in, say, the possession of a backbone, a skull and limbs; it may be inferred by analogy that the extinct animal resembled the living animal in the possession of such other organs as a heart, a nervous system and organs of sense, muscles and tissues as are possessed by the latter. Two plants resemble each other in the essential parts of their flowers as well as in their leaves, branches and general appearance; therefore they will resemble also in their fruit. This is an inference by analogy. The fruit of one, as in the case of mango trees, may resemble that of another in form and size; but the quality of one may be quite different from that of the other. A bird and a bat have flying organs and other superficial resemblances. What is true of one is not true of the other, because they differ in the more important and fundamental organs in virtue of which they belong to different classes of the backboned animals, the bat being a mammal.

§ 4. (iii) Inductions of Succession or Causation.

Whenever a causal connection can be established between two phenomena, the relation between them gives rise to a valid induction. According to Mill the cause of a phenomenon is the sum of positive and negative conditions which being realised, the phenomenon invariably and unconditionally follows. Take the phenomenon of lighting a match by rubbing it on the surface of the box. The cause of the phenomenon of a lighted match is the friction of a chemically prepared end of a little stick on the chemically prepared surface of the match-box. Here the phenomenon is a luminous flame, and its cause is a positive condition, viz. the presence of certain circumstances and a negative condition, viz.

the absence of certain other circumstances. The positive condition consists of (1) a small stick duly prepared with necessary chemicals, (2) a chemically prepared surface of the match-box, (3) the skilful rubbing of the prepared end of the stick on the prepared surface, (4) the usual atmospheric condition. The negative condition consists of (1) the absence of damp in the atmosphere, in the stick-end and on the prepared surface, (2) the chemical preparation of the end of the stick and of the surface of the box not having in the meantime disappeared or been badly affected, (3) the rubbing not being on a part of the surface which has, by previous use or otherwise, lost its chemical properties. In other words, the cause of the flame is the assemblage of the positive conditions, all counteracting circumstances being absent.

These positive and negative conditions being fulfilled, the effect, namely, a luminous flame at one end of the stick, follows. The effect will not follow if any of the conditions which must be absent are present or if any of the positive conditions are absent. In the example given above, the rubbing would be popularly regarded as the cause of the phenomenon of lighting the match, though the chemical preparations of the stick and the surface are essential conditions of it. If the match is not lighted by the rubbing, the cause would popularly be considered to be the dampness of the air—the absence of which is one of the negative conditions. There is thus great difference between Mill's definition of the term Cause and its popular use. It has been pointed out that it is desirable to use the term Cause neither in Mill's sense nor in the popular sense, but in a sense intermediate between the two, that is, to mean the sum of the minimum conditions which being given, the effect invariably follows. In the example of the lighting of a match, the minimum conditions would be the chemical preparations of the stick and the surface plus the rubbing. These therefore should be usually regarded as the

cause of the phenomenon, the conditions being severally regarded not as a cause of it but as essential or indispensable to it.

§ 5. Mill regards the Law of Causation as the most certain law of Nature. Though he proves it by the method of Simple Enumeration, he thinks that there are no exceptions to it. All our experience, he says, confirms it. If there were any exceptions, our experience would discover it. Scientific men in dealing with a phenomenon proceed on the assumptions (1) that it has another phenomenon or group of phenomena for its antecedents, and (2) that the same antecedents will always produce the same effect. When they are unable to trace a phenomenon to its antecedent, they try to find out the phenomena with which it co-exists and also the phenomena which it resembles. The phenomena coexisting with the one under investigation will give rise to an induction or law of coexistence. In the example given above, the phenomena coexisting with the given phenomena of a lighted match are (1) the heat of the flame, (2) the smoke which escapes from the flame, (3) the rise in temperature of the air in contact with the flame, (4) the smell of the gases produced, etc. All these phenomena coexist in the effect and form parts of it. In a scientific inquiry, therefore, the object is to find the phenomena connected with a given phenomenon under investigation. The connected phenomena may be those coexisting with the given phenomenon and thus suggesting the class of phenomena to which it belongs, or those preceding it. In the former case a law of Coexistence and along with it a law of Resemblance will be discovered, and in the latter case a law of Succession which may be shown to be a law of Cause and Effect. It may be further shown that the phenomena coexisting are joint effects of a common Cause; and a law of Coexistence may be thus resolved into a law of Succession or Causation. The light and heat of the flame are

coexisting phenomena in the above example and are joint effects of the common cause, namely, the rubbing of the stick-end on the surface of the box.

The object of scientific investigation is to discover and prove the laws of Coexistence, Resemblance and Succession and to resolve them into laws of Causation among phenomena.

§ 6. Inductions have been distinguished into (1) Perfect and (2) Imperfect. A perfect induction is the inference of a general proposition after the examination of every individual belonging to a class. An imperfect induction is the inference of a general proposition after the examination of some individuals of a class. In the former, what is already known to be true of every individual belonging to a class is inferred of the whole class. In the latter what is known to be true of some individuals of a class is inferred of the whole class. Every known planet is observed to go round the sun: the inference, from the particular observations of these known planets, of the general proposition "all the known planets go round the sun," is a perfect induction, while the inference from the same observations of the general proposition "all planets go round the sun" is an imperfect induction. The former applies only to the planets that are known to us, while the latter applies also to those that may be discovered afterwards. "All metals conduct heat and electricity" is an imperfect induction as it applies to the elements known now to be metals and also to those that may hereafter be discovered to be metals, while the proposition "all the metals conduct heat and electricity" would be a perfect induction if it was inferred after the examination of all individual known metals and applied to them only; but it would be an imperfect induction, if it was inferred from the examination of some individual metals and applied to all the known metals. According to Mill an induction proper is an inference from the known to the unknown,

from the observed to the unobserved on the ground of the Uniformity of Nature. In the case of a perfect induction there is no such transition; there is, according to him, no inference at all. The so-called general proposition or induction is merely a summary of the individual observations. A perfect induction is not, according to him, a generalisation but merely a summary statement of the particulars already known individually. It is true that, in a perfect induction, there is no inference from some to all individuals of a class as in the case of an imperfect induction, but in both there is reliance on the Uniformity of Nature. That all the known planets will continue to move round the sun, that all the known metals will retain all their properties and continue to conduct heat and electricity, are inferred on the ground of the Uniformity of Nature and the perfect inductions may therefore claim to be recognised as inductions. In an imperfect induction, the principle of the Uniformity of Nature forms the ground of the transition from some to all as well as from the present to the future. Both perfect and imperfect inductions therefore depend for their validity on the Uniformity of Nature. The inference that the sun will in future daily rise in the east and set in the west is an induction drawn from the instances of its rising in the east and setting in the west. That a particular tree will in future years bear fruit of the same quality as it has done in the past is an induction founded on the Uniformity of Nature. It will bear fruit of the same quality if the conditions of the soil and the atmosphere continue the same as before, but if either of these conditions changes, the fruit of the tree may be of a much inferior quality. The induction is drawn from the instances of its bearing fruit of the same quality in past years. We thus have inductions in regard to an individual thing as well as a class.

CHAPTER VI.

The Methods of Induction.

§ 1. The different methods by which inductions are established are

(1) The Method of Simple Enumeration,

(2) The Methods of Elimination by which the unnecessary circumstances can be excluded and the necessary or relevant circumstance or circumstances discovered as cause of a given effect or as effect of a given cause. Mill's methods of experimental inquiry are methods of elimination.

Given a phenomenon, it is required to find other phenomena connected with it. The phenomena connected with it may be (1) those coexisting with it, (2) the antecedents or the consequents of it. In the second case, the phenemenon is given as an effect and it is required to find its cause; or it is given as a cause and it is required to find its effect. In the first case, it is required to find the phenomena coexisting with the given phenomenon, no distinction being made between antecedents and consequents.

§ 2. The Method of Simple Enumeration.

The Method of Simple Enumeration is the most ancient as well as the most popular method of induction. The earliest knowledge of mankind in regard to natural phenomena and human life and society is founded on this method. Children acquire their knowledge of things from a very early period of their life by this method. They instinctively believe that what has happened once will happen again under the same circumstances. The burnt child dreads the fire.

The Method of Simple Enumeration consists in inferring a general proposition from instances of a phenomenon, observed in Nature or in human affairs, without any systematic attempt at the analysis of the phenomena and the discovery of the true cause or the true effect by the elimination of the circumstances which are irrelevant or unnecessary. It does not seek and consider the negative instances, that is, instances in which the phenomenon is absent. It simply states the instances in which the phenomenon occurs and generalises on the basis of those instances. The validity of the induction thus drawn depends on the number of the instances observed and the possibility or probability of there being contrary instances. If the induction, as in the case of the proposition "all animals are mortal," is confirmed by all our experience, it is accepted as true. If, on the contrary, it is contradicted by some contrary instances as in the case of the proposition "all plants have flowers," it is modified and limited in extent.

§ 3. Symbolical Statement of the Method of Simple Enumeration.

(1) Let the given phenomenon be represented by *A* and the phenomena or circumstances coexisting with it by *B*, *C*, *D*, *E* &c. Observe the phenomenon A in a number of instances in which it happens, and let the instances be represented as follows without any distinction of antecedents and consequents:—

A B C D E
A F G H E
A K L M E

These instances prove that *A* and *E* are connected: both may be joint effects of a common cause or one may be cause and the other effect. Other instances of *A* may be observed and the connection of *A* with *E* confirmed by them.

(2) Observe the phenomenon *A* in a number of in-

stances in which the antecedents are distinguished from the consequents, the former being represented by capital letters and the latter by small ones.

Let the instances be as follows:—

A B C	*p q r*
A B D	*p q s*
A D E	*p s t*
etc.	etc.

Here *A* is the antecedent given and *p* the consequent found connected with it in the above instances: *p* is therefore an effect of *A*. In the reverse case, in which the phenomenon given is a consequent, represented by *p*, the above instances will indicate that *A* is a cause of the effect *p*.

The instances in both cases may be multiplied without any attempt at elimination or exclusion of unnecessary or irrelevant consequents in one case, and of unnecessary or irrelevant antecedents in the other case. Other instances may be observed and may be represented as follows:—

A B K	*p q n*
A C L	*p r n*
A E H	*p t v*
etc.	etc.;

A standing as before for the given antecedent phenomenon in one case and *p* for the given consequent phenomenon in the other case.

§ 4. Concrete Examples.

(1) The given phenomenon is the presence of flowers in plants; it is required to find the other phenomena with which it is connected. Taking some instances of higher plants, it is found that the phenomenon of flowers is connected with the phenomena of the presence of leaves, branches, roots in the same plant, and the induction drawn from them is that all these phenomena co-

exist in all higher plants. Do they coexist in all plants? The instances of lower plants such as ferns and mosses, which are flowerless, show that those phenomena do not coexist in all plants. Are they connected causally? Are flowers cause or effect of roots, branches and leaves? Are they all joint effects of a common cause? These questions come under the investigation of a Botanist. The popular answer would be that all those parts of a plant are connected and produced alike by the plant which is the result of the germination of a seed. At first only a little root and two leaves are produced from the seed under proper conditions of soil, air, light and heat, and this little plant grows and gradually produces all the parts including the trunk.

(2) The given phenomenon is the backbone of an animal: it is required to find out the other phenomena with which it is connected in an animal.

Taking some instances of animals having a backbone, it is found that it is connected with a head, limbs, a heart, a nervous system and organs of sense, etc., and the induction is drawn from these instances that these parts coexist in all animals possessing a backbone. On further observation it is found that all fishes, all amphibia (frogs, etc.), all reptilia (snakes etc.), all birds and all mammalia including man have these parts coexisting in every individual. Are they causally connected? Is the backbone the cause or effect of the others? Are they joint effects of a common cause? These questions come under the investigation of the comparative Anatomist and Embryologist. The popular answer would be on the same lines as in the preceding example.

(3) Given the yellow colour of a piece of gold as the phenomenon under investigation. The connected phenomena would be the chemical and physical (other than the colour) qualities of gold possessed by each sample. They all coexist with the yellow colour in every molecule of gold. They are, as we say, inherent in it

and cohere in the substance gold. Are they causally connected? Can we say some are cause and some others are effect? Are they all joints effects of a common cause? These questions belong to the sciences of Physics and Chemistry.

(4) Take the phenomenon of a lighted spirit lamp given as an antecedent and observe the instances of its heating and expanding water, oil, mercury, etc., in a flask. Here the antecedent phenomena are the lighted spirit lamp, a liquid (water, oil, mercury), and a flask. The consequent phenomena are the liquid expanded in the glass flask over the lighted spirit lamp. From these instances, it may be inferred that the expansion of the liquid is the effect of the given antecedent, the lighted spirit lamp. Taking the expansion of the liquid as the given phenomenon, it may be inferred that the lighted spirit lamp is the cause of it. The induction is that heat expands liquids. Another phenomenon common to all the consequents, which could be easily observed by touch or by a thermometer, is the rise of temperature of the liquid. This is also an effect of the given cause, the lighted spirit lamp. The induction is that heat raises the temperature of liquids. What is the relation of the two effects, expansion and rise in temperature of the liquids, to each other? Are they coexistent or one antecedent and the other consequent? They take place simultaneously and appear to be joint effects of the same cause, heat.

§ 5. According to Mill the Laws of Thought, the Axioms of Geometry, the Principles of the Uniformity of Nature and of Causation, etc., are generalisations from experience by this method. They are regarded and accepted as universally true because there are no exceptions to them, because they are confirmed by all our experience. (1) The axiom that things equal to the same thing are equal to one another, is a generalisation from the innumerable instances in which we find that two things equal to

the same thing are equal to each other. (2) The Law of Thought, that a thing cannot have two contradictory attributes—that A cannot at the same time be B and not-B—is a generalisation from our experience that B and not-B are never found coexisting in one and the same subject, that the presence of B in a thing means the disappearance of not-B, and the presence of not-B the disappearance of B, that the two ideas B and not-B cannot in any case exist together. (3) The proposition that every phenomenon is preceded by another phenomenon, is a generalisation by this method. In all instances of phenomena in the external world, we find that a phenomenon has always another phenomenon for its antecedent. The induction is confirmed by all our experience of physical phenomena. It forms the basis of Mill's Law of Causation which he applies not only to all physical but also to mental phenomena.

CHAPTER VII.

(2) The Methods of Elimination.

§ 1. Mill's Methods of Experimental Inquiry are methods by which the cause of a given effect or the effect of a given cause is determined by the elimination of the irrelevant or unnecessary circumstances.

Let p be a given effect; it is required to find its cause.

Observe or produce by experiment an instance in which p occurs. Analyse the antecedent circumstances of the instance and represent them by A, B, C. In this case the cause of p may be the aggregate of the three circumstances $A\ B\ C$ or a combination of any two of them, $A\ B$, or $A\ C$, or $B\ C$, or any single circumstance A, or B, or C. The effect p may be of such a nature that it requires the interaction of all the three circumstances to produce it. On this supposition the cause of p will be the sum of the circumstances $A\ B\ C$. Or the effect p may require the interaction of only two circumstances and these may be either $A\ B$, or $A\ C$, or $B\ C$. Or the effect p may be produced by one circumstance only, and this may be A, or B, or C.

§ 2. In Nature we have instances of phenomena which are produced in the several ways noted above. For example, the sensation of light is produced by (1) a luminous object, (2) the organ of sight, and (3) the faculty of seeing. These three are necessary to produce any sensation of light. They must therefore be taken together and regarded as the cause or indispensable conditions of the effect—a sensation of light.

The growth of a plant by the germination of a seed is brought about by the interaction of agents such as the

soil, the moisture, the heat and light of the sun, the air of the atmosphere, the vitality of the seed itself—all of which should be taken together and regarded as cause or indispensable conditions of the phenomenon. In chemical actions, the elements producing a compound must jointly be regarded as the cause of the compound produced. The effects in these cases are quite unlike the effects of the individual circumstances or agents which jointly by their interaction produce them. There are, again, instances of phenomena in which the effect is the sum of the effects of the several antecedents which jointly produce it. In these cases each of the several antecedents produces the same kind of effect. The simplest example of such cases is the action of two forces on a body in the same direction, producing an effect which is the sum of the effects of the two forces. Two forces acting on a body in opposite directions will produce an effect which is the difference of the effects of the two forces. Two forces acting on a body at an angle will move the body in a direction which can be determined by the parallelogram of forces, the sides representing the two forces and the diagonal representing the resultant motion as the effect. A liquid may be heated by three different lamps of spirit, kerosine and mustard oil; the effect will be the sum of the effects produced by them, as each antecedent produces the same sort of effect, namely, the expansion and rise in temperature of the liquid. A room may be lighted by ten candles, the effect produced by each candle being a certain degree of luminosity; the total effect is the sum of the effects produced by all. In these cases, some of the antecedents may be removed without changing the nature of the effect.

In determining the causes of phenomena, it must, therefore, be remembered that the cause of a phenomenon may consist either of antecedents which jointly produce an effect which is quite different from the effect of each of them, or of antecedents which produce an effect which

is of the same nature as the effect of each of them. Mill calls the former Chemical or Heteropathic Intermixture of effects and the latter Mechanical or Homogeneous Intermixture of Effects. "In the Mechanical, the effect is the sum of the effects of the several antecedents; and the cause is equivalent to the effect." To this fact is due the idea that the cause and its effect are identical or equal. This is true in the causation of the phenomena of Physics where all the causes produce some sort of motion. This is hardly true in Chemistry, and much less in Biology and not at all in Psychology. In Chemistry the weight of the constituent substances taken together is found, by experiment, to be equal to the weight of the compound produced; and this is the only property in which they agree: in other respects the properties of the compound are quite different from those of the constituent substances. For example, the properties of water are quite different from those of hydrogen or oxygen. In Biology, the form and the qualities of the plant produced by the germination of the seed are quite different from those of the antecedent agents. So are also the forms and qualities of the leaf, the flower, the fruit, etc., which the plant gradually produces, from those of the ingredients of the soil and atmosphere, under the influence of the sun. In Psychology, the sensation of light is quite unlike the antecedents. So are also other sensations unlike their antecedents. "The mental phenomena are quite different from their physical antecedents. As biological phenomena are to a great extent determined by the vitality and nature of the seed, so are also the phenomena of the mind determined mainly by the nature and faculties of the mind itself. "Some philosophers maintain that they are entirely due to the mind itself and that the relationship between the mental phenomena and the corresponding physical antecedents is merely one of concomitance, while others hold that there is interaction between them—that in sensation, for

example, a psychical phenomenon is produced in the mind by some physical antecedents and that in voluntary action, on the other hand, a physical phenomenon is produced by some mental antecedents. In any case the mind itself must be regarded as an essential condition of every mental phenomenon.

§ 3. Given an effect *p*, it may be produced by several distinct causes. Heat, for example, may be produced by mechanical action such as rubbing or by chemical action such as the union of hydrogen with chlorine. Motion may be produced by steam or by electricity. Death may be caused by various diseases. It may be brought about by the failure of the heart, by the decay of the lungs or by the stoppage of the function of some vital part of the brain. Sensations may be produced by external objects or by subjective causes, that is, by changes in the brain or in the organs of sense of the organism itself. Germination of a seed may be brought about by artificial means as well as by natural causes. There are thus many instances of phenomena where an effect of the same kind is produced by causes which are distinct and separable from one another. In one instance the effect is produced by one cause and in another instance the same effect is produced by another cause quite distinct from the first. There is thus in Nature the fact of a plurality of distinct causes of an effect of the same kind. This fact must be recognised and allowed for in applying the Methods of Elimination for the discovery and proof of the cause of a given effect.

It has been pointed out that these cases of a plurality of causes of a given effect arise from our overlooking the collateral circumstances of the given effect. If all the consequent circumstances are taken into consideration, it would be found that the same consequents are always produced by the same antecedents, that the given effect *p* is only one of the many circumstances which together form the complex effect of the many

antecedent circumstances which together constitute the complex cause. Death by heart failure has, for example, collateral circumstances which are different from those of death by the decay of lungs or by some other cause. The phenomenon of death is common to the several instances of dying; but the other circumstances connected with it are different in each case. The phenomenon of the production of heat is likewise common to the different instances in which heat is produced by mechanical or by chemical action, but the other circumstances connected with it are different in each case. So the fact of a plurality of causes of the same effect is due to our overlooking all the circumstances of a complex consequent, except that one which is given as an effect and which is present not only in the instance under consideration but in many other instances.

§ 4. The Canons of Mill's Methods of Experimental Inquiry.

Mill's experimental methods of inquiry proceed on the assumption of the truth of the two fundamental principles of Causation and Uniformity of Nature. They are conducted according to certain laws or rules which he calls the Canons of the Methods. In Appendix D of my Text-book of Deductive Logic, I have shown that the canons follow from the two laws of Causation and Uniformity of Nature.

The law of Causation implies the two propositions—(1) every phenomenon has a cause and (2) the cause of a phenomenon is the invariable or, as Mill says, the unconditionally invariable antecedent of the phenomenon. The law of Uniformity of Nature means that (3) the same cause or antecedent will, under the same circumstances, produce the same effect or consequent. From the second proposition of the first law follows (4) "whatever antecedent can be left out, without prejudice to the effect, or whatever antecedent can be present, without the effect being present, can be no part of the cause"; (5)

"when an antecedent can not be left out, without the consequent disappearing, such an antecedent must be the cause or part of the cause"; (6) "an antecedent and a consequent rising and falling together in numerical concomitance are to be held as cause and effect"; (7) "if two or more instances of a phenomenon under investigation have only one circumstance in common, that circumstance is the cause (or effect) of the phenomenon"; and (8) "if an instance where a phenomenon occurs and an instance where it does not occur have every circumstance in common except one, that one occurring only in the first, the circumstance present in the first and absent in the second, is the cause or a part of the cause of the given phenomenon."

The proposition marked (7) is the canon of the Method of Agreement. The proposition marked (8) is the canon of the Method of Difference. The proposition marked (6) is the canon of the Method of Concomitant Variations. The joint Method of Agreement and Difference is really a method of agreement both in presence and absence. The canon consists of the two propositions marked (7) and (5). The Method of Residues is an experimental method which is aided by the Method of Deduction. The propositions marked (4) and (5) are the rules respectively for the elimination of irrelevant and unnecessary circumstances and for the detection and proof of the cause of a given effect.

CHAPTER VIII.

The Method of Agreement and the Joint Method.

I. Method of Agreement.

§ 1. The canon of the Method is:—If two or more instances of a phenomenon under investigation have only one circumstance in common, that circumstance is the cause (or effect) of the phenomenon.

Mill proceeds on the assumption that the phenomenon under investigation, if it is an effect, has only one distinct cause and that it is not the result of an intermixture of effects of several antecedents. He further assumes that the antecedents and the consequents of an instance of the phenomenon under investigation may be analysed into circumstances which are simple and separable from one another.

Let the phenomenon under investigation be an effect represented by p and two instances of it be represented as follows:—

(1) $A\ B\ C\ \ldots\ \ldots\ p\ q\ r$
(2) $A\ D\ E\ \ldots\ \ldots\ p\ s\ t$

where $A\ B\ C$ are the simple antecedents and $p\ q\ r$ the simple consequents of one instance; and $A\ D\ E$ the simple antecedents and $p\ s\ t$ the simple consequents of the other. The given phenomenon p as an effect is present among the consequents of both the instances, $q\ r$ being the collateral circumstances in one and $s\ t$ in the other.

From (1), $D\ E$ cannot be any part of the cause of p, according to proposition (4) marked in Chapter VII, Section 4, that is, the rule, "Whatever antecedent can be

left out, without prejudice to the effect, can be no part of the cause."

From (2), *B C* cannot be any part of the cause of *p*, according to the same rule.

From (1) and (2), it may therefore be inferred that the one simple antecedent circumstance, *A*, which is common to both the instances, is the cause of *p*, according to the canon of this Method.

This inference is true (1) if *p* has only one distinct cause and (2) if it is not the result of an intermixture of the effects of several antecedents.

§ 2. Plurality of Causes and Intermixture of Effects.

(1) If *p* has a plurality of distinct causes, there may be two other instances of it represented as follows:—

(1) *F G H* *p l m*
(2) *F I K* *p n o*

From those two instances it may be inferred that *F* is a cause of *p*.

There may be other instances where *p* is produced by other distinct causes.

(2) If *p* is the result of an intermixture of the effects of several antecedents, then a combination of any two antecedents *A B*, or *A C* or *B C*, or the aggregate of all the three antecedents *A B C* may be the cause of *p* in the first instance of the first example. In the second instance of the same example, *A D*, or *A E*, or *D E*, or *A D E*, any one of these combinations of antecedents, may be the cause of *p*. In such a case the Method of Agreement is not applicable. The cause of the phenomenon cannot be found by this method. There will then be no one circumstance common to the antecedents of all the instances of the phenomenon. Suppose *p* is the result of the intermixture of the effects of *A B*, then there may be two instances of it as follows:—

(1) $\overline{AB}$ *C D* *p q r*
(2) $\overline{AB}$ *E H* *p s t*

From (1) and (2), it may be inferred that $\overline{AB}$ is the cause of p. If p has a plurality of distinct combinations of antecedents as causes, then there may be instances of it as follows:—

(1) $\overline{FG}\ H\ K$	$p\ l\ m$
(2) $\overline{FG}\ I\ L$	$p\ n\ o$

From (1) and (2), it may be inferred that $\overline{FG}$ is the cause of p.

Thus p may have a plurality of causes also in the case of its being the result of an intermixture of effects. Mill points out that when the phenomenon under investigation is the result of a homogeneous or mechanical intermixture of effects of several antecedents, the investigation of its cause or causes should be conducted according to the Deductive Method.

It may be pointed out that when the phenomenon under investigation is the result of an heterogeneous or chemical intermixture of effects of several antecedents, its cause may be investigated by the Method of Agreement. The one common circumstance among the antecedents of the phenomenon will then be not simple but compound or complex, consisting of several antecedents, as for example, in the case of the sensation of light, or the germination of a seed, or the production of water from the union of hydrogen and oxygen.

When p has several distinct causes, for example, A, F, etc., it may be found on careful scrutiny of A, F etc., that each of them is complex, consisting of two or more simpler circumstances; and that among these there is some circumstance which is common to them all. This will be then a case of intermixture of effects, as p will be produced by the circumstance common to A, F, etc., plus the circumstance or circumstances in which they differ. It may be that the common circumstance is a merely nominal antecedent and cannot be regarded as

the cause of p, though such an inference would be justified by the Method of Agreement. Whether such an antecedent is the cause or not may be verified by the Method of Difference.

§ 3. Examples:

1. Cold applied to water in an iron vessel freezes it. Cold applied to cocoanut oil in a glass bottle freezes it. Therefore cold is the cause of freezing. The phenomenon under investigation is that of freezing. Cold is the one common circumstance present in the antecedents of the two instances of the phenomenon and is therefore its cause.

2. Heat applied to the frozen water in the iron vessel melts the water. Heat applied to the frozen cocoanut oil in a glass bottle melts the oil. Therefore heat is the cause of melting.

3. Attention paid to a perception leads to its retention in memory. Attention paid to an idea leads to its retention in memory. The phenomenon under investigation is that of retention in memory of a mental state. Attention is the common circumstance in the antecedents of the two instances of the phenomenon. Therefore attention is the cause of the phenomenon of retention.

4. Prove the following propositions by the Method of Agreement:—

(i) Heat is the cause of the expansion of material bodies.

(ii) A luminous object is a condition of the sensation of light.

(iii) An organ of sense is a condition of a sensation.

(iv) Contiguity is a cause of the association of mental phenomena.

(v) Resemblance is a cause of the assimilation of mental phenomena.

II. The Joint Method of Agreement and Difference or the Method of Agreement both in Presence and Absence.

§ 4. The Canon of the Method is :—" If two or more instances of a phenomenon under investigation have only one circumstance in common, while two or more instances in which the phenomenon does not occur have nothing in common except the absence of that circumstance, the circumstance in which the two sets of instances differ is the cause (or effect) of the phenomenon, provided both the sets of instances belong to the same department of inquiry "

Let p, an effect, be the phenomenon under investigation and the instances in which it occurs be represented as follows:—

I. Positive instances—

$A\ B\ C$	$p\ q\ r$
$A\ D\ E$	$p\ s\ t$
$A\ F\ G$	$p\ u\ v$

Let the instances in which p does not occur be represented as follows:—

II. Negative instances—

$C\ H\ F$	$r\ w\ x$
$B\ D\ K$	$q\ y\ z$
$E\ G\ M$	$i\ h\ n$

From the first set of instances it may be inferred by the Method of Agreement that "A is the cause of p," under conditions which have been discussed under that Method. This inference is confirmed by the second set of instances in which both A and p are absent, and which therefore show according to rule 4 given in Chapter VII, Section 4, that the antecedents B, C, D, E, F, G which are present in one or other of the three positive instances as well as of the three negative instances, without the effect p being present in the latter, cannot be the cause of p. The inference drawn from the first set is confirmed by the second set, by excluding all the antecedents except A in the positive instances from the category of the

cause of p. There is thus agreement of A with p in both presence as in the positive instances and in absence as in the negative instances. The negative instances should be selected from the department of inquiry, to which p belongs; and if they could be made exhaustive in that department, the inference would be of great value. In the symbolical example given above the common letters show that the two sets of instances are taken from the same department. But it would still be true only under the conditions discussed under the Method of Agreement.

§ 5. Examples:

Prove the following propositions by the Joint Method:—

(1) Cold is the cause of the freezing of water.
(2) Heat is the cause of the melting of ice.
(3) A luminous object is an indispensable condition of a sensation of light.
(4) The faculty of hearing is an indispensable condition of a sensation of sound.
(5) Attention is the cause of the retention of a mental state.
(6) Contiguity is a cause of the association of mental phenomena.
(7) Resemblance is a cause of the assimilation of mental phenomena.

CHAPTER IX.

III. The Method of Difference.

§ 1. The Canon of the Method is :—"If an instance where a phenomenon occurs and an instance where it does not occur have every circumstance in common except one, that one occurring only in the first, the circumstance present in the first and absent in the second is the cause, or a part of the cause, of the given phenomenon."

Let the given phenomenon be p and the two instances be represented as follows :—

(1) $A\ B\ C\ \ldots\ \ldots\ p\ q\ r.$
(2) $B\ C\ \ldots\ \ldots\ q\ r.$

Here the two instances, in the first of which p occurs and in the second it does not occur, have every circumstance in common, except A : the circumstance A, occurring only in the first and absent in the second, is, therefore, the cause or a part of the cause of p.

Taking the example of lighting a match, p would be the flame, $q\ r$ its collateral effects, A the rubbing and $B\ C$ the other antecedent circumstances, the stick and the surface. The two instances would be (1) the antecedents and consequents after rubbing when the effect, the flame, is produced, and (2) the antecedents and consequents before rubbing. They agree in every circumstance except one, namely, the circumstance of rubbing; this circumstance is therefore the cause of the flame. This is a popular statement of the circumstances; and the rubbing is regarded as the cause of the effect. In a scienti-

fic statement it would be necessary to follow the sequence of the phenomena more closely. The rubbing produces a little heat which brings about a union of the chemical substances of the stick and the surface, producing some gaseous products and more heat. This heat raises the temperature of the gases, causing the kindling of the stick and the flame. Between the rubbing and the flame there are several phenomena connected as cause and effect. The flame is a remote effect of the rubbing. The immediate effect of the rubbing is the production of the little heat which again acts as an antecedent and produces along with other antecedents certain effects which bring about the kindling of the stick. Scientifically, therefore, the inference which can be drawn from the two instances is that the rubbing is the cause of the little heat which is necessary to bring about the union of the chemical substances and is only a remote condition of the flame. This concrete example shows that the two instances should be, strictly speaking, in immediate succession—that there should be no lapse of time and no other links between them, and that they should be identical in all respects except in the antecedent circumstance which produces the effect; and that this effect is the immediate change brought about by the antecedent and not the remote consequent which may be produced by the effect acting as an antecedent along with other antecedents as a cause. The flame is such a remote consequent and is produced by the conjoint action of several antecedents. Each of these antecedents may be proved by the Method of Difference to be a part of its cause.

In the case of a sensation of light, the physical conditions are (1) a luminous object and (2) the eyes with the nerves and brain, and the mental conditions are (3) the faculty of vision, and (4) attention; and it can be proved by the Method of Difference that each of these conditions is a part of the cause of the phenomenon of the

sensation of light. When all these conditions are fulfilled a sensation of light is produced; and when any one of these conditions is wanting the sensation is not produced.

In the case of the germination of a seed, it may be proved by the Method of Difference that the conditions of the soil, the moisture, the heat and light of the sun, and the atmospheric air are parts of the cause of the phenomenon. Each of these may be alternately supplied to, and removed from, the seed and the effect observed. The difficulty lies in making the experiments, securing all the conditions required.

In the examples given above, the effect p is the result either of a single antecedent as in the case of the heat produced by the rubbing or of an heterogeneous intermixture of the effects of several antecedents. In the former case the single antecedent is the cause, and in the latter each of the several antecedents is a part of the cause of the phenomenon. In the symbolical statement of the latter, A would be a complex group of antecedents, and p an effect which is produced by the conjoint action of the several antecedents. The positive instance in which the effect is present would be represented as follows:—

$$\overline{ABC}\ \ D\ E \qquad .. \quad p\ q\ r$$

where three different antecedents conjointly produce the effect p, $D\ E$ being the other antecedents and $q\ r$ being the other consequents. A negative instance in which p is absent would be as follows:—

$$\overline{BC}\ D\ E \quad .. \qquad .. \quad q\ r$$

proving that A is a part of the cause of p, or as follows:—

$$\overline{AC}\ \ D\ E \quad .. \qquad .. \quad q\ r$$

proving that B is a part of the cause of p, or as follows:—

$$\overline{AB} \quad D\,E \;\ldots \qquad \ldots \quad q\,r$$

proving that C is a part of the cause of p.

These three negative instances with the one positive instance would respectively prove that each of the three antecedents A, B, C is a part of the cause of p, while a single negative instance $D\ E\ \ldots\ q\ r$, with the positive instance $A\ B\ C\ D\ E \ldots . p\ q\ r$ would prove that the three antecedents A, B, C are jointly the cause of p.

§ 2. If p is the result of a homogeneous intermixture of the effects of several antecedents, then its cause cannot be found by the Method of Difference. Let p be the sum of the effects of A and B, $q\ r$ the collateral consequents and $C\ D$ the collateral antecedents, then the instance in which p is present will be represented as follows :—

$$\overline{AB} \quad C\,D \;\ldots \qquad \ldots \quad p\,q\,r$$

If A is absent in the second instance and B is present in it, p will be still partly present in it as p is the result of the sum of the effects of both A and B. Thus the negative instance in which p is entirely absent will not be available ; and the Method of Difference will be inapplicable in such cases. Mill points out that such cases should be investigated by the Deductive Method.

If $A\ B$ be taken together as a complex or compound antecedent consisting of two conditions or circumstances, then $A\ B$ may be inferred to be a cause of p from these two instances :—

$$\begin{array}{lll} \overline{AB}\ CD \;\ldots & \ldots & p\,q\,r \\ \quad CD \;\ldots & \ldots & \quad q\,r. \end{array}$$

But in order to find the effect of A, it would be necessary to deduct from p the effect of B ; and this would be done according to the rules of the Deductive Method: that is, the effect of each of the two antecedents being known by previous inductions, the effect of their interaction would be determined by the Deductive Method.

§ 3. Examples :

(1) Heat applied to water in a vessel raises its temperature and is therefore the cause of the rise of its temperature. The two instances are (1) heat applied to water in a vessel raising its temperature and (2) water in a vessel without any heat being applied to it. The phenomenon under investigation is the rise of temperature of the water. It is present in the first and absent in the second. The one circumstance, application of heat, present in the first and absent in the second is, therefore, the cause or a part of the cause of the rise of temperature. The two instances agree in everything except in the one circumstance, application of heat.

(2) An external object is an indispensable condition of a sensation. This proposition can be proved by the Method of Difference in the case of the several senses, (1) by having an appropriate external object affecting the special organ of sense of a person and his attending to it in one instance (positive) and (2) by removing the object in the other instance (negative) so that his organ of sense may not be affected by it. From two such instances it may be inferred that the object is a cause or a part of the cause of the sensation.

(3) By similar observations or experiments it may be inferred that attention on the part of a person is an essential condition of the consciousness of a sensation. His organ of sense, say, that of hearing, may be affected by a sound ; but he will not be conscious of it if he is deeply attending to something else, that is, the consciousness of a sensation of sound depends on his attending to it. A student intently listening, say, to a lecture in the class, does not often hear the bell. A person deeply absorbed in study does not hear even when he is called loudly.

(4) Prove the following propositions by the Method of Difference :—

(i) Cold is the cause of freezing.

(ii) Heat is the cause of the melting of ice.
(iii) A special organ of sense is an indispensable condition of sensation.
(iv) Attention and repetition are the causes of the retention of a mental state.
(v) The development of a faculty depends on its exercise.
(vi) Life is the cause of the organisation of a body.
(vii) Mind is the cause of feelings.

CHAPTER X.

IV. The Method of Concomitant Variations.

§ 1. The Canon of the Method is :—"An antecedent and a consequent rising and falling together in *numerical concomitance* are to be held as cause and effect."

A more accurate statement would be as follows :—

Whenever an antecedent and a consequent of a phenomenon vary together in a certain manner either directly or inversely, that is, both increasing together or one increasing and the other decreasing, the antecedent is either the cause, or a part of the cause, of the consequent.

A more comprehensive statement would be as follows :—

Two circumstances of a phenomenon varying together in a certain manner, that is, either increasing together or one increasing and the other decreasing, are causally connected, that is, connected as cause and effect or as joint effects of a common cause.

Let A be an antecedent and p a consequent, the two rising and falling together as represented in the following instances :—

I.

(1)	$A\ B\ C\ \ldots$	$\ldots\ p\ q\ r$
(2)	$2\ A\ B'\ C'\ \ldots$	$\ldots\ 2\ p\ q'\ r'$
(3)	$3\ A\ B''\ C''$	$\ldots\ 3\ p\ q''\ r''$
	etc.	etc.

In these three instances, p increases as A increases: A is therefore the cause of p. The collateral circumstances of p change with the connected antecedents of A.

There may, however, be instances where they remain unchanged as represented below :—

II.

(1)	$A\ B\ C$	$p\ q\ r$
(2)	$2\ A\ B\ C$.	.. $2\ p\ q\ r$
(3)	$3\ A\ B\ C$.	. $3\ p\ q\ r$

In the two sets of instances given above, a consequent and an antecedent rise together. When the antecedent is doubled, the consequent is doubled. The relation between the two may not always be so simple. All that the Canon requires is that the two should vary together according to some definite numerical relation.

The Method of Concomitant Variations may be regarded as an extension of the Method of Agreement in the case of the first set of instances, and as an extension of the Method of Difference in the case of the second set of instances. The inference drawn by this method is therefore subject to the conditions discussed under those two methods.

It is sometimes stated that the Method of Concomitant Variations is a method for determining the quantitative relation between an antecedent and a consequent, when a qualitative relation between them has been established by the Methods of Agreement and Difference. These two methods may prove that a particular antecedent is a cause or a part of a cause of a particular consequent and the Method of Concomitant Variations may then find and prove the quantitative relation between them.

The Method of Concomitant Variations can be applied to determine the qualitative or the quantitative relations between two or more circumstances of a phenomenon, when they cannot be separated from it and when the Method of Difference cannot therefore be applied. In the first four examples given below, the gravity, inertia,

attraction and weight of a body cannot be separated from it and the Method of Difference cannot therefore be applied to determine the effect of any one of these circumstances. In such cases the Method of Concomitant Variations is of great use and is the only method which can be applied to them. It may then be regarded as an extension of the neglected Method of Simple Enumeration by combined observation and experiment.

§ 2. Examples:

(1) The attraction of a material body for another varies directly as its mass; that is, as the mass of a body increases, its attraction for another body also increases: therefore, the antecedent, the mass of a body, and the consequent, its attraction for another, are cause and effect.

(2) The weight of a body varies directly as its mass on the same part of the surface of the earth, that is, as the mass of a body increases, its weight also increases in the same place: the antecedent, the mass of the body, and its weight are cause and effect.

(3) The weight of a body varies inversely as its distance from the centre of the earth. In this case, as the distance of the body increases its weight decreases; and as the distance decreases the weight increases: therefore the weight of a body and its distance from the centre of the earth are causally connected.

(4) The attraction of a body for another varies inversely as its distance from that body; that is, as the distance increases the attraction decreases: therefore the attraction of a body for another and its distance from that body are causally connected.

In these examples, a certain antecedent and a certain consequent vary in a certain manner: in some cases as one increases, the other increases; in some others as one increases, the other decreases: the inference drawn by the Method of Concomitant Variations is that the antecedent and the consequent are causally connected,—that

the antecedent is the cause or a part of the cause of the consequent.

If the circumstances of the phenomenon of the attraction of a material body for another are regarded as coexisting and therefore incapable of being distinguished as antecedents and consequents, then the circumstances of its mass, its attraction, its weight and its distance from the centre of the earth, varying together in a certain manner, may be the joint effects of a common cause. According to the law of gravitation, two material bodies attract each other inversely as the square of the distance between them. The attribute of being a material body and the power of attraction are regarded as coexistent and cannot be distinguished into cause and effect—it is not known that the one is the cause and the other the effect; but both may be the joint effects of a common cause. The weight of a body is due to the attraction of the earth and varies according to its distance from the centre of the earth. This follows from the law of gravitation which is a generalisation from the phenomena of coexistence of the two attributes, inertia and gravitation, found in all material bodies. The law is established by the Method of Simple Enumeration. It is found by observation that whatever has inertia has the power of gravitation; and a material body is defined by the possession of these two attributes. The mass of a body is defined by its inertia, that is, the resistance it offers to movement. The Method of Concomitant Variations proves that the mass and the power of attraction of a body for another vary together, that the weight of a body varies with the attraction of the earth upon it, and that the latter decreases as the distance of the body increases from the centre of the earth.

(5) A room 10 feet by 8 feet is lighted by one candle. It is then lighted by two candles. It is then lighted by three candles—all of the same size and quality. The differences in the intensity of the lighting of the room

in the three cases are carefully observed by a scientific instrument called Photometer. It is found that the intensity increases with the number of candles lighted. It may, therefore, be inferred by the Method of Concomitant Variations that the number of candles lighted is the cause or part of the cause of the degree of illumination in a particular case. By the Method of Difference, it may be proved that the candle light is the cause of the illumination of the room, by two instances—one in which a candle is lighted and the other in which there is no candle light in the room; and by the Method of Concomitant Variations, a quantitative relation between the quantity of candle light and the degree of illumination of the room may be established.

(6) Heat a certain quantity of water in a vessel by a lamp and observe the rise of its temperature by a thermometer. Heat the same quantity of water in the same vessel for the same period of time by two lamps of the same dimensions and quality as in the first case and repeat the experiment with three lamps of the same dimensions and quality, observing in each case the rise of temperature by a thermometer. The observations show that the temperature increases with the number of lamps, that is, with the increase of heat applied to the vessel. It is, therefore, inferred by the Method of Concomitant Variations that the quantity of heat applied is the cause, or a part of the cause, of the rise of temperature of the water in the vessel. As in example (5), it may be first proved by the Method of Difference that heat is the cause of the rise of temperature; and then a quantitative relation between the heat applied and the degree of rise in the temperature may be established.

(7) The retention of an idea in memory varies with its repetition. It varies also with the degree of attention paid to it. The more an idea is repeated, the greater is its retention. The repetition of an idea is therefore a cause or a part of the cause of the retention of the idea in

memory. The greater the attention paid to the idea each time it is repeated, the greater is its retention. The retention of an idea in memory depends therefore on attention and repetition.

(8) The development of a faculty or function varies with its exercise. The more a faculty is used, the greater is its development. The use or exercise of a faculty is therefore a cause or a part of the cause of its development.

(9) The size and structure of the brain of an animal vary with its intelligence. The greater its intelligence, the larger and the more complex in structure its brain. The intelligence and the brain of an animal are therefore causally connected.

(10) The organisation of an animal varies with its life. The higher the life, the more complex is its organisation. The life of an animal and its organisation are therefore causally connected.

CHAPTER XI.

V. The Methods of Residues.

§ 1. If a part of the consequents of a phenomenon can be shown by deduction from previous inductions to be the effect of some of the antecedents, then the remaining part of the consequents is the effect of the remaining antecedents.

Let $A\ B\ C$ be the antecedents and $p\ q\ r$ the consequents of a phenomenon. If $q\ r$ can be shown, by deduction from B and C, to be their effect, then p is the effect of A. The distinction between this Method and the Method of Difference is that in the latter, the part $q\ r$ is shown to be the effect of $B\ C$ by an experiment or observation, while in the Method of Residues, it is deduced from the laws of B and C, established by previous observations and experiments

(1) $A\ B\ C$ . . . . $p\ q\ r$
(2) $B\ C$ . . . . $q\ r$
$\therefore$ p is the effect of A.

In the Method of Residues $q\ r$ is deduced from B and C. In the Method of Difference $B\ C \ldots . q\ r$ is known by experiment and observation. In the Method of Residues the phenomenon under investigation is the result of an observation or experiment; but the proof that p is the effect of A is deductive, depending on the axiom that if equals be taken from equals, the remainders are equal. It is assumed that there is a sort of equality between the antecedents as the cause and the consequents as the effect and that a part of the ante-

cedents being shown to be equal to a part of the consequents, the remaining consequents are equal to the remaining antecedents.

Mill gives the Canon of the Method as follows :—

"Subduct from any phenomenon such part as is known by previous inductions to be the effect of certain antecedents, and the residue of the phenomenon is the effect of the remaining antecedents."

§ 2. Examples :

(1) A room 10 ft. by 8 ft. is lighted jointly by two candles of a certain quality and size and a lamp, and the degree of illumination produced by them is observed by a photometer. The degree of illumination of the room, which is produced by a single candle of the same quality and size, has been ascertained by previous experiments. The effect of the two candles can be deduced from the recorded result of the experiments. Subduct this effect from the degree of illumination of the room produced jointly by the lamp and the two candles. Then it may be inferred by the Method of Residues that the remaining degree of illumination is the effect of the remaining antecedent, the lamp. This inference may be verified by an actual experiment with the lamp—that is, the room may be lighted with the lamp only and the degree of illumination produced by it, as observed by a photometer, should agree with the calculated result.

(2) The effect of the heat of a lamp may similarly be determined by heating a certain quantity of water in a vessel by the lamp and two candles, if the heating power of a candle of a certain quality and size has been ascertained by previous experiments.

(3) The existence of cohesion as a separate form of attraction from gravitation can be determined by this Method. The law of gravitation explains many movements of material bodies, but cannot account for the phenomena of the cohesion of the molecules in the same body.

(4) The existence of chemical affinity, vitality and mind as causes is established by this Method. The phenomena of Nature are of different kinds and cannot be accounted for by one and the same cause. Take the chemical phenomena: many of them are due to heat, electricity, light and other physical forces; but there are some, for example, the union of hydrogen and oxygen according to a certain proportion by weight, and of other elements with one another, generalised under the law of chemical combination, that cannot be accounted for by them. It is therefore inferred by this Method that the phenomena which are not the effects of physical forces are caused by a special force called chemical affinity. The phenomena of living things likewise lead to the inference of vitality as a special cause of those phenomena of life which cannot be shown to be the effects of chemical and physical forces. The argument has been applied also to prove the existence of mind as a separate and special cause of those phenomena, for example, of feelings and thoughts of living beings, which are called psychical and which can not be shown to be the effect of vital, chemical or physical forces.